CEE 141 - FALL 2018
HOMEWORK 2 SOLUTION

NOTE: Perform calculations for LRFD method ONLY.

I. Complete the following problems from the textbook:
Chapter 5 – Compression Members:

5-3

W24×370, A992, $L_{eff} = 15\,ft$, $A_g = 109\,in.^2$ $I_y = 1160\,in.^4$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29000)(1160)}{(15(12))^2} = 10{,}200^k$$

$$P_y = F_y A_g = 50(109) = \underline{5450}^k$$

∴ column will yield at 5450^k

5-5

DETERMINE EFFECTIVE LENGTH FOR $P_{cr} = P_y$

W12×40, A992, $A_g = 11.7\,in.^2$ $I_y = 44.1\,in.^4$

$$P_y = F_y A_g = 50(11.7) = 585^k$$

For $P_{cr} = \frac{\pi^2 EI}{(KL)^2} = 585$,

$$KL = \sqrt{\frac{\pi^2 EI}{P_y}} = \sqrt{\frac{\pi^2(29000)(44.1)}{585}}$$

$KL = 147\,in. \Rightarrow 12.3\,ft.$

5-9

W14x48 $I_x = 484\ in.^4$, $I_y = 51.4\ in.^4$ $KL_x = 20'$

$$P_{cry} = \frac{\pi^2 E (51.4)}{(12)^2 (KL_y)^2} = P_{crx} = \frac{\pi^2 E (484)}{(12(20))^2}$$

$$KL_y = 20\sqrt{\frac{51.4}{484}} = 6.52\ ft.$$

5-12

W14x120, A992, $KL = 40\ ft.$ $A_g = 35.3\ in.^2$

$r_y = 3.74$

$$\frac{KL}{r_y} = \frac{40(12)}{3.74} = 128 > 4.71\sqrt{\frac{E}{F_y}} = 113$$ ∴ ELASTIC Eq E3-3

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 (29000)}{(128)^2} = 17.5\ ksi$$

$$F_{cr} = 0.877 F_e = 0.877(17.5) = 15.3\ ksi$$

$$P_n = F_{cr} A_g = 15.3(35.3) = 540\ k$$

LRFD

$$\phi P_n = 0.9(540) = 486\ k$$

5-18

HSS $4 \times 4 \times \frac{3}{8}$, A500 GR B, $KL_y = 10$ ft, $KL_x = 15$ ft.

$F_y = 46$ ksi, $A = 4.78$ in.^{2}, $r_x = r_y = 1.47$ in.

$$\frac{KL_x}{r_x} = \frac{15(12)}{1.47} = 122^*, \quad \frac{KL_y}{r_y} = \frac{10(12)}{1.47} = 81.6$$

$$\frac{KL}{r} = 122 > 4.71\sqrt{\frac{E}{46}} = 118 \quad \therefore \text{ INELASTIC, E3-2}$$

$$F_e = \frac{\pi^2 E}{(122)^2} = 19.2 \text{ ksi}$$

$$F_{cr} = 0.658^{(46/19.2)}(46) = 16.9 \text{ ksi}$$

$$P_n = F_{cr} A = 16.9(4.78) = 80.8^k$$

LRFD

$$\phi P_n = 0.9(80.8) = 72.7^k$$

II. Also, answer the following problems:

1. A W18 × 119 is used as a compression member with one end fixed and the other end pinned. The length is 12 feet. What is the available compressive strength if A992 steel is used?
   a. Use AISC Equation E3-2 or E3-3. Compute both the design strength for LRFD
   b. Use Table 4-22 from Part 4 of the *Manual.* Compute both the design strength for LRFD

Solution:

(a) $\frac{KL}{r} = \frac{0.8(12 \times 12)}{2.69} = 42.83$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(42.83)^2} = 156.0 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

Since $KL/r = 42.83 < 113.4$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/156.0)}(50) = 43.72 \text{ ksi}$$

$$P_n = F_{cr}A_g = 43.72(35.1) = 1535 \text{ kips}$$

Design strength $= \phi_c P_n = 0.90(1535) = 1382$ kips $\qquad \underline{\phi_c P_n = 1380 \text{ kips}}$

(b) From *Manual* Table 4-22, for $KL/r = 42.83$ and $F_y = 50$ ksi,

$\phi_c F_{cr} = 39.33$ ksi (by interpolation)

$\phi_c P_n = \phi_c F_{cr} A_g = 39.33(35.1) = 1380$ kips $\qquad \underline{\phi_c P_n = 1380 \text{ kips}}$

2. A 20-foot long column is pinned at the bottom and fixed against rotation but free to translate at the top. It must support a service dead load of 110 kips and a service live load of 110 kips. Select a W12 of A992 steel. Use the column load tables.

Solution:

$KL = 2.0(20) = 40$ ft

(a-1) $\quad P_u = 1.2D + 1.6L = 1.2(110) + 1.6(110) = 308$ kips

From the column load tables for $KL = 40$ ft, a W12 × 120 has a design strength of 339 kips.

<u>Use a W12 × 120</u>

3. An HSS 10 × 6 × 5/16 with *Fy* = 46 ksi is used as a column. The length is 15 feet. Both ends are pinned, and there is support against weak axis buckling at a point 6 feet from the top. Determine the design strength for LRFD.

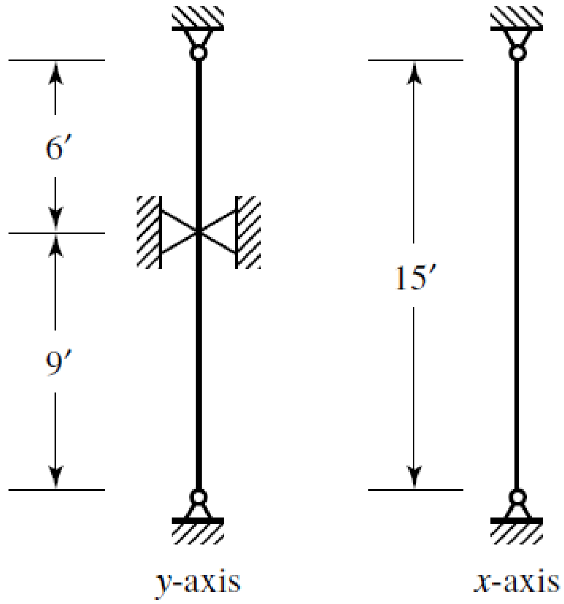

Figure 1

Solution:

For an HSS 10 × 6 × 5/16, $A_g = 8.76 \text{ in.}^2$, $r_x = 3.66$ in., $r_y = 2.47$ in., and there are no slender elements (see table that follows *Manual* Table 1-12).

$$\frac{K_xL}{r_x} = \frac{15 \times 12}{3.66} = 49.18, \qquad \frac{K_yL}{r_y} = \frac{9 \times 12}{2.47} = 43.72$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(49.18)^2} = 118.3 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{46}} = 118.3$$

Since $KL/r = 49.18 < 118.3$, use AISC Eq. E3-2:

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(46/118.3)}(46) = 39.09 \text{ ksi}$$

$$P_n = F_{cr}A_g = 39.09(8.76) = 342.4 \text{ kips}$$

$$\phi_c P_n = 0.90(342.4) = 308.2 \text{ kips}$$

$\underline{\phi_c P_n = 308 \text{ kips}}$

4. Use A992 steel and select a W shape.

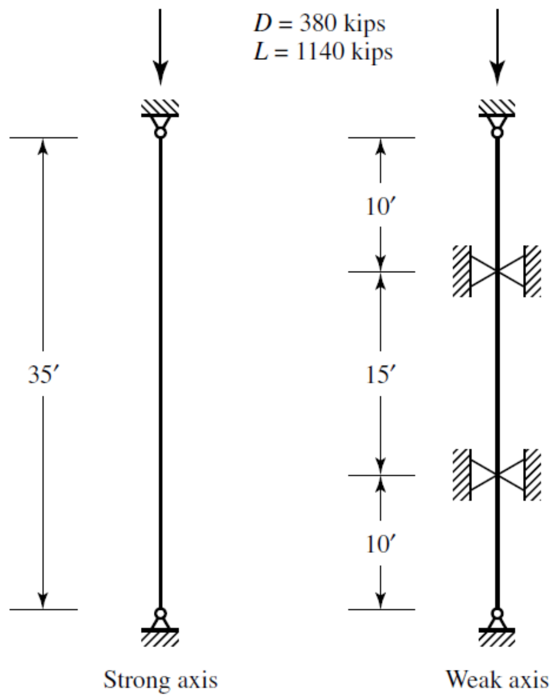

Figure 2

Solution:

$K_xL = 35$ ft, $\quad K_yL = 15$ ft

(a) $P_u = 1.2D + 1.6L = 1.2(380) + 1.6(1140) = 2280$ kips

From the column load tables for $KL = 15$ ft, there are no W8 or W10 shapes with enough strength. Try a W12 × 230 :

$\phi_c P_n = 2450$ kips for $KL = 15$ ft

$$\frac{K_xL}{r_x/r_y} = \frac{35}{1.80} = 19.44 \text{ ft} > 15 \text{ ft}$$

For $KL = 19$ ft, $\phi_c P_n = 2150$ kips < 2280 kips (N.G.)

Try a W12 × 252:

$$\frac{K_xL}{r_x/r_y} = \frac{35}{1.81} = 19.34 \text{ ft} > 15 \text{ ft}$$

For $KL = 20$ ft, $\phi_c P_n = 2280$ kips $= P_u$ (OK)

Investigate W14 shapes: Try a W14 × 211. $\phi_c P_n = 2420$ kips for $KL = 15$ ft

$$\frac{K_xL}{r_x/r_y} = \frac{35}{1.61} = 21.74 \text{ ft.} > 15 \text{ ft}$$

For $KL = 20$ ft, $\phi_c P_n = 2160$ kips < 2280 kips (N.G.)

Try a W14 × 233:

$$\frac{K_xL}{r_x/r_y} = \frac{35}{1.62} = 21.60 \text{ ft} > 15 \text{ ft}$$

For $KL = 21.60$ ft, $\phi_c P_n = 2300$ kips > 2280 kips (OK)

The W14 × 233 is the lightest W shape (in the column load tables) that will work.

Use a W14 × 233

5. A column for a multistory building is fabricated from ASTM A588 plates as shown in Figure P4.9-5. Compute the nominal axial compressive strength based on flexural buckling (do not consider torsional buckling). Assume that the components of the cross section are connected in such a way that the section is fully effective.

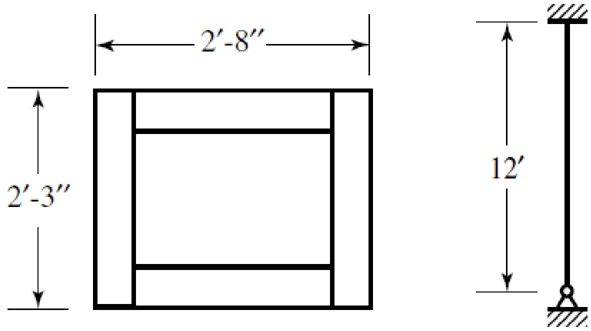

Figure 3

Solution:

$$F_y = 42 \text{ ksi}$$

$$A = 32(27) - 20(15) = 564 \text{ in.}^2$$

$$I_{\min} = I_x = \frac{1}{12}(32)(27)^3 - \frac{1}{12}(20)(15)^3 = 4.686 \times 10^4 \text{ in.}^3$$

$$r_{\min} = r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{46860}{564}} = 9.115 \text{ in.}$$

$$\frac{KL}{r} = \frac{0.8(12 \times 12)}{9.115} = 12.64$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(12.64)^2} = 1791 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{42}} = 123.8 > 12.64$$

$$\therefore\ F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(42/1791)}(42) = 41.59 \text{ ksi}$$

$$P_n = F_{cr}A_g = 41.59(564) = 2.346 \times 10^4 \text{ kips}$$

$\underline{P_n = 23{,}500 \text{ kips}}$